Primal-Dual, a novel approach to optimisation in treatment planning

An alternate automated planning process for heavy ion beam therapy

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Pseudo-Algorithm for the Forward Model

1. Fix the beam angles; initialise:
   - Beam Shape
   - Beam Energy
2. Compute all dose profiles.
3. Calculate optimal profile.

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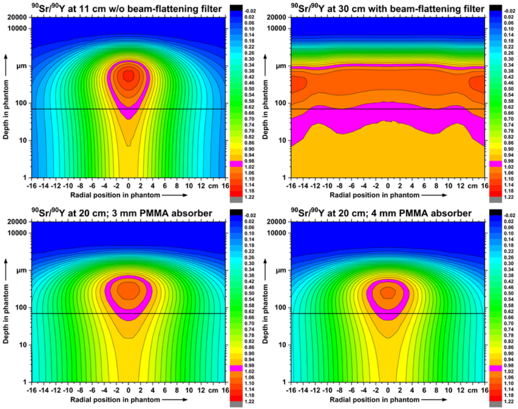

Image Source: Energy-reduced beta radiation fields from 90 Sr/ 90 Y for the BSS 2, May 2020, Journal of Instrumentation, R. Behrens

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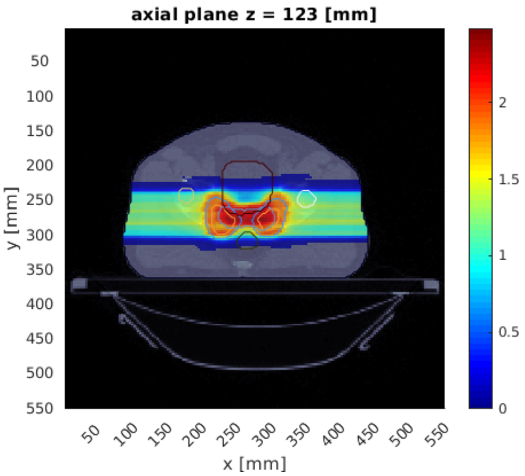
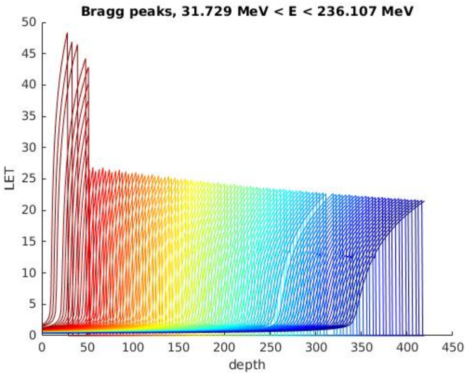


Source code: matRad courtesy of the German Cancer Rsearch Center (DKFZ)

Computational complexity of current psuedo-algorithms



Computational Complexity

No. of forward solves =
No. possible beam shapes ×
No. possible beam energies ×
No. variations for Robustness

Current Methodology

For $i = 1, \dots, N$ let $\mathcal{D}_i$ denote the precomputed dose profiles

$$\min_{a_i \geqslant 0} \left\| \sum_{i=1}^{N} a_i \mathcal{D}_i - \mathcal{D}_T \right\|_{\mathrm{PTV}}^2 + \sum_{n=1}^{N} w_n \| a_i \mathcal{D}_i \|_{\Omega_n}^2 \qquad (1)$$

Alternate Methodology

Let u denote the particle density

$$\min_{u,f} \| \mathcal{D} u - \mathcal{D}_T \|_{\mathrm{PTV}} + \sum_{n=1}^{N} w_n \| \mathcal{D} u \|_{\Omega_n}^2 + \alpha \| f \|^2 \qquad (2)$$

subject to u satisfying Boltzmann transport equation with admissible inflow.

Pseudo-Algorithm for the Primal-Dual system

1. Start with a given initial configuration.

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6. Repeat steps 2-5.

Advantages of the Primal-Dual system: Certainty of Convergence

Advantages

1. Mathematical certainty of convergence
2. Significantly fewer forward solves
3. Accounting for uncertainties in the model

Properties of Primal-Dual

1. Existence of solutions to the stationary problem.
2. Uniqueness of solutions to the stationary problem.
3. Stability of solutions of the time-dependent problem.

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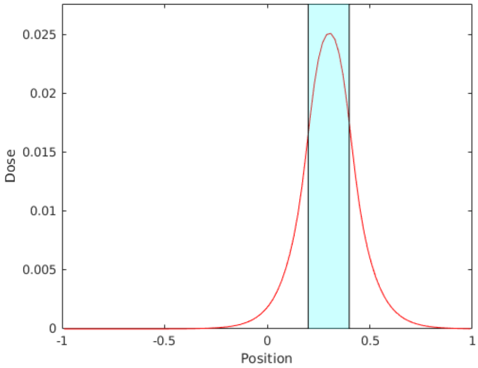

Figure: Optimised solution to the Kolmogorov equation.

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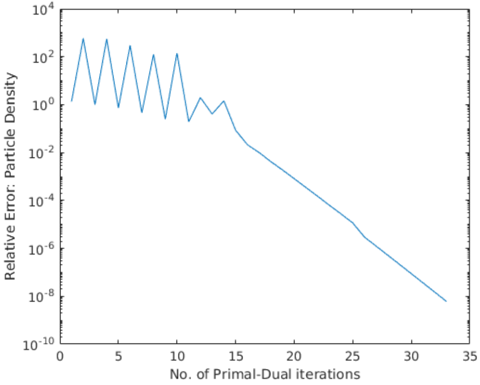

Figure: Relative error between each Primal-Dual iteration.

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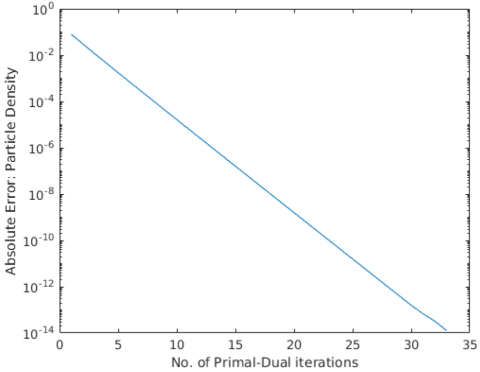

Figure: Absolute error between each Primal-Dual iteration.

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Perturbed system

Let $\epsilon(x)$ be a random function then consider

$$\max_{\mathbb{P}(\epsilon)<0.95} \min_{u,f} J(u,f)$$

subject to

$$\frac{\partial u}{\partial t} + \Omega \cdot \nabla_{\mathbf{x}} u = \sigma_a u - \iint_{I\times\mathbb{S}^2} \sigma_s u d\Omega' dE'$$

$$u(t,\mathbf{x},E,\Omega) = f(t,\mathbf{x}+\epsilon(\mathbf{x}),E,\Omega)$$

Thank you for listening

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